

1. The function $f(x)$ is defined by $f(x) = \begin{cases} (x-3)^2, & x \leq 3 \\ 1 - \frac{a}{x}, & x > 3 \end{cases}$.

If $f(x)$ is continuous at $x = 3$, determine the value of a . [4]

2. Solve the equation $\log_4 x + 15 \log_x 4 = 8$ [5]

3. An arithmetic sequence has the first term a and the common difference d . The tenth term is 69 and the sum of the first 30 terms is four times the sum of the first 10 terms. Find the values of a and d . [5]

4. Show that $\int_1^e x^2 \ln x dx = \frac{1}{9}(2e^3 + 1)$. [5]

5. Sketch the graph of $y = |x^2 - 4|$. [2]

Hence, find the solution set of the inequality $|x^2 - 4| < 3x$. [4]

6. (a) Given that $A = \int_{-1}^1 \left(\frac{1}{1+e^{-x}} \right) dx$, show that the estimate of A obtained by using the trapezium rule with three ordinates is 1. [4]

(b) By means of the substitution, $u = e^x$, show that the estimate obtained in (i) is correct. [4]

7. The functions f and g are defined as

$$f: x \rightarrow 3 \ln x, x > 0$$

$$g: x \rightarrow \sqrt{x}, x \geq 0.$$

(i) Sketch the graph of f , and give a reason why the inverse function exists. [3]

(ii) Find f^{-1} , and state its domain. [3]

(iii) Find the composite function gf^{-1} , and state its range. [3]

8. One of the factors of the polynomial $p(x) = 8x^4 - 4x^3 + ax^2 + bx - 3$ is $2x - 1$.
When $p(x)$ is divided by $(x + 2)$, the remainder is 135. Determine the values of the constant a and b . [3]

Factorise $p(x)$ completely. Show that the equation $p(x) = 0$ has only two real roots, and find both these real roots. [7]

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9. Find the equation of the locus of $P(x,y)$ such that it is equidistant from the lines $4y = 3x + 8$ and $y = 2$. [3]

The straight line l_1 is perpendicular to $4y = 3x + 8$ and passes through $A(0,12)$.
 If l_1 cuts the locus of P at M , show that $AM:OM = 5 : \sqrt{13}$ where O is $(0,0)$ [4]

Find the angle between the line l_1 and the locus of P . [3]

10. (a) Find the possible values of k if the matrix $\begin{pmatrix} 2 & k+3 & 1 \\ 2k & 4 & -3 \\ 2 & 5 & 1 \end{pmatrix}$ is singular. [3]

(b). Given the matrices $A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 3 & 4 \\ 3 & 4 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & -6 & 1 \\ -5 & 3 & 1 \\ 2 & 0 & -1 \end{pmatrix}$. $A^{-1} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 2 & 4 & 3 \end{pmatrix}$

Find AB and deduce A^{-1} .

Hence, solve the following simultaneous equations using matrices.

$$\begin{aligned} x + 2y + 3z &= 15 \\ x + 3y + 4z &= 21 \\ 2x + 4y + 3z &= 15 \end{aligned} \quad [8]$$

11. Given that $f(x) = \frac{2}{(x-1)(x+3)}$, express $f(x)$ in partial fractions. [3]

Hence, expand $f(x)$ in ascending powers of x up to and including the term in x^3 . [7]

Determine the set of values of x for which the expansion is valid. [2]

12. (a) If $y = (1-x)(e^{-x}) - 8$, show that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y + 8 = 0$ [4]

(b) Sketch, on the same coordinate axes, the graphs $y = e^{-x}$ and $y = \frac{8}{1-x}$.
 Show that the equation $(1-x)(e^{-x}) - 8 = 0$ has a root that lies between -2 and -1. [5]

Use the Newton-Raphson method with the initial estimate $x_1 = -1$ to estimate the root correct to three decimal places. [6]

*****END OF QUESTION PAPER*****

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